

range, the ratio seems to be almost constant. This implies therefore that there exists to a good approximation a reduced  $\theta_D$  curve of the form

$$\theta_D = F(V)f(T/\theta_D). \quad (13)$$

Since  $-\partial \ln \theta / \partial \ln V = 2.4$  and is very nearly constant over the whole volume range investigated, we can write  $F(V) \propto V^{-2.4}$ . These results mean that a Gruneisen equation of state describes the data quite well over a wide range of volumes and temperatures.

As already mentioned, it should be possible to calculate values of  $\theta_0$  from the elastic constants of the solid at the density of interest. Although we do not know the complete elastic constants of solid helium we do know its compressibility as a function of volume. From this alone it is possible to calculate an approximate value of  $\theta_0$  by making some assumption about how Poisson's ratio varies with volume. The simplest relation of this kind implies that

$$\theta \propto (a/\beta m)^{\frac{1}{2}}, \quad (14)$$

where  $a$  is the lattice parameter,  $\beta$  the compressibility, and  $m$  the mass of the atom. In table 12 we compare the experimental values of  $\theta_D$  evaluated at the same reduced temperature ( $\theta_D/T = 18$ ) with the corresponding values of  $(a/\beta m)^{\frac{1}{2}}$  for both helium isotopes.

TABLE 12. COMPARISON OF  $\theta_D$  FROM SPECIFIC HEATS WITH  $\theta$  ESTIMATED FROM THE COMPRESSIBILITY

The values of  $\theta_D$  are taken at the same reduced temperature  $T/\theta_D = 18$ . The values in brackets are extrapolated from slightly higher reduced temperatures.

$V$ (cm <sup>3</sup> /mole)	$\theta_D$ (°K)	$\theta_D(\beta m/V^{\frac{1}{2}})^{\frac{1}{2}}$
	<sup>3</sup> He	
12.57	101.2	500
13.33	89.3	497
13.56	85.8	496
14.16	78.0	495
14.98	68.1	491
15.72	60.5	491
16.71	(51.9)	482
16.87	(50.9)	483
17.02	(49.6)	480
	<sup>4</sup> He	
11.77	99.9	499
12.22	91.9	509
14.55	62.0	526
16.25	(46.6)	510

(Instead of  $a$ , the lattice parameter, we have written  $V^{\frac{1}{2}}$ .) The last column of the table gives the product  $\theta_D(\beta m/V^{\frac{1}{2}})^{\frac{1}{2}}$ . Although for <sup>3</sup>He it shows small systematic changes and for <sup>4</sup>He some bigger, apparently random, changes, this product remains rather constant over the whole range. We may therefore conclude that the volume dependence of  $\theta$  reflects quite closely the volume dependence of  $\beta$ . By putting in a reasonable value for the constant of proportionality in equation (14) (cf., for example, Mitra & Joshi 1961) it is found that the magnitude of  $\theta$  so calculated is similar to the values of  $\theta_0$  indicated by the extrapolations in figures 7 and 8. Since the theory is only approximate this agreement is not very significant but it does perhaps suggest that if there is a low temperature anomalous contribution to the specific heat as found by Heltemes & Swenson it is *not* due to the lattice vibrations.